Topic: Divisibility, congruences, binomial theorem. Operation, rings and fields. Zero divisors, subring, ideal, generated ideal. Factor ring, ring homomorphism, kernel and image. Direct sum of rings. Number theory in integral domains (unique factorization, principal ideal ring, Euclidean ring).

58. True or false?
(a) $(k, n) = 1 \Rightarrow n \mid \binom{n}{k}$.
(b) $n \mid \binom{n}{k} \Rightarrow (k, n) = 1$.

59. Determine the sum and product of all non-zero elements in $\mathbf{Z}_p$ where p is a prime.

60. For which positive integers m do 0 and the zero-divisors form an ideal in $\mathbf{Z}_m$?

61. True or false?
(a) The intersection of two non-zero ideals in a ring is a non-zero ideal.
(b) The intersection of two non-zero ideals in a zero-divisor free ring is a non-zero ideal.

62. Let $\varphi : \mathbf{Z}_{100} \to \mathbf{Z}_{100}$ be defined by $\varphi(x) = cx$. Find all $c \in \mathbf{Z}_{100}$ for which φ is a ring homomorphism.

63. For which rings R is the map $\varphi : R \oplus R \to R$ defined by $\varphi((r, s)) = r + s$ a ring homomorphism?

64. Let $S = \{f : \mathbf{R} \to \mathbf{R}\}$ be the ring of all real functions under the usual addition and multiplication of real functions and $g(x) = \begin{cases} 0, & \text{if } x = 2021; \\ \pi, & \text{if } x \neq 2021. \end{cases}$
Show that $S/(g)$ is a field.

65. Let F be a field. Prove: $F[x]/(g)$ is a field iff g is irreducible over F.

66. Show that the ring of finite decimal fractions is a UFD, moreover it is a PID, and even a ED.

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