BSM Fall 2021

Introduction to Abstract Algebra **AL1**

Instructor: Dr. Róbert FREUD

Text: Peter J. Cameron: Introduction to Algebra (Oxford University Press 1998), Chapters 1, 2, and 3 + handouts

Prerequisite: None

Abstract algebra grasps some essential common features of seemingly very different mathematical objects and provides a framework to investigate their structure in general. The topic of the course is an introduction to the theory of two such important algebraic structures: rings and groups. We will immediately realize that we already know many rings and groups of various forms from our earlier mathematical studies, and their unified treatment will provide a better understanding of their intrinsic properties. We shall see new examples and discover interesting relations through solving lots of exercises and problems of various difficulty. Also, we shall apply the theory to answer some questions from seemingly remote areas, such as analysis, number theory, combinatorics, or even social sciences, e.g.:

- Can the real function $f(x) = x$ be written as the sum of two periodic functions?
- Which positive integers can be represented as the sum of two squares?
- At most how many positive integers up to n can have the property that their pairwise sums are all distinct?
- In how many ways can we color 4 squares red in a 5×5 square grid where two colorings count the same if a rotation or a reflection can transform them into each other?
- How were the marriage rules set at certain native South American tribes?

Topics:

Rings. Subrings, ideals, factor rings, homomorphism, direct sum. Number theory in rings: unique factorization theorem, principal ideal domains, Euclidean rings, Gaussian integers, two squares theorem. Field extensions, finite fields, application to Sidon sets.

Groups. Subgroups, cyclic groups, order of an element. Cosets, Lagrange's theorem, Cauchy's theorem. Normal subgroups, conjugacy, factor groups, homomorphism, direct product. Permutation groups, Cayley's theorem, Burnside's lemma. Structure of groups of small size.